Problem Ping-pong

C++ header `ping-pong.h`

Three friends, Anna, Bob and Charlie, play ping-pong. Ping-pong is a two-player game, so while two of them play the third simply watches. After a game is over, the player who won stays at the table, and then plays with the person who was watching. For example, if Anna beats Bob, then she plays the next match with Charlie.

One day, their informatics teacher asked them the following question. How many ways could the three have played so that Anna wins *exactly* a matches, Bob *exactly* b matches and Charlie *exactly* c matches? Because this can be a very large number, the teacher is satisfied with the answer modulo $10^9 + 7$.

We now explain when we consider two ways of playing different. For a way of playing P, let $\mathrm{Win}(P)$ be the list of players who win each game and $\mathrm{Watch}(P)$ be the list of players who watch each game. Then, two ways of playing P and Q are considered to be different if there is a match i for $1 \le i \le a + b + c$ where $\mathrm{Win}(P)_i \ne \mathrm{Win}(Q)_i$ or $\mathrm{Watch}(P)_i \ne \mathrm{Watch}(Q)_i$.

This question seems pretty difficult for them. Can you help them?

Implementation details

You should implement the following function.

```
int solve(int a, int b, int c);
```

This function should return the answer for the given values of a, b and c. It will only be called *once* per execution by the committee's grader.

You may use the following function while implementing your solution.

```
int combinations(int n, int k);
```

This will return in constant time $\binom{n}{k} \bmod 10^9 + 7$, i.e. the binomial coefficient corresponding to n, k, or the number ways of choosing k objects from among n objects, all modulo $10^9 + 7$. Note that the parameters n and k need to satisfy $0 \le k \le n \le 5\,000\,000$.

Remember to include the header `ping-pong.h`!

Sample grader behaviour

The sample grader will read three integers, a, b and c from standard input. It will then call `solve`(a, b, c), and output the returned value to standard output. The input/output files below work for this grader.

Restrictions

- $0 \le a, b, c \le 1\,000\,000$.

#	Points	Restrictions
1	9	$a + b + c \leq 12$
2	4	$a + b \leq 20, c = 0$
3	7	$a \leq 1\,000, b \leq 1\,000, c = 0$
4	11	$c = 0$
5	21	$1 \leq a, b, c \leq 100$
6	22	$1 \leq a, b, c \leq 1\,000$
7	26	No further restrictions.

Examples

Input file	Output file
1 1 1	6
2 2 2	24
286 191 90	789937023

Explanations

In order to compactly represent all the different scenarios that could happen, we will use the following notation. If in a game player x beats player y, we denote this by $x \to y$. Anna is player a, Bob is player b and Charlie is player c. We will then show a string of games as a list of such statements: for example, if Anna beats Bob, Charlie beats Anna and then Bob beats Charlie, we represent this by $a \to b, c \to a, b \to c$.

First Example. There are only 6 possible scenarios:

1. $a \to b, c \to a, b \to c,$
2. $b \to a, c \to b, a \to c,$
3. $a \to c, b \to a, c \to b,$
4. $c \to a, b \to c, a \to b,$
5. $b \to c, a \to b, c \to a,$
6. $c \to b, a \to c, b \to a.$

Second Example. There are 24 possible scenarios:

1. $a \to b, a \to c, b \to a, c \to b, c \to a, b \to c,$
2. $a \to b, c \to a, b \to c, a \to b, c \to a, b \to c,$
3. $a \to b, c \to a, b \to c, b \to a, c \to b, a \to c,$
4. $a \to b, c \to a, c \to b, a \to c, b \to a, b \to c,$
5. $b \to a, b \to c, a \to b, c \to a, c \to b, a \to c,$
6. $b \to a, c \to b, a \to c, a \to b, c \to a, b \to c,$
7. $b \to a, c \to b, a \to c, b \to a, c \to b, a \to c,$
8. $b \to a, c \to b, c \to a, b \to c, a \to b, a \to c,$
9. $a \to c, a \to b, c \to a, b \to c, b \to a, c \to b,$

10. $a \to c, b \to a, b \to c, a \to b, c \to a, c \to b,$
11. $a \to c, b \to a, c \to b, a \to c, b \to a, c \to b,$
12. $a \to c, b \to a, c \to b, c \to a, b \to c, a \to b,$
13. $c \to a, b \to c, a \to b, a \to c, b \to a, c \to b,$
14. $c \to a, b \to c, a \to b, c \to a, b \to c, a \to b,$
15. $c \to a, b \to c, b \to a, c \to b, a \to c, a \to b,$
16. $c \to a, c \to b, a \to c, b \to a, b \to c, a \to b,$
17. $b \to c, a \to b, a \to c, b \to a, c \to b, c \to a,$
18. $b \to c, a \to b, c \to a, b \to c, a \to b, c \to a,$
19. $b \to c, a \to b, c \to a, c \to b, a \to c, b \to a,$
20. $b \to c, b \to a, c \to b, a \to c, a \to b, c \to a,$
21. $c \to b, a \to c, a \to b, c \to a, b \to c, b \to a,$
22. $c \to b, a \to c, b \to a, b \to c, a \to b, c \to a,$
23. $c \to b, a \to c, b \to a, c \to b, a \to c, b \to a,$
24. $c \to b, c \to a, b \to c, a \to b, a \to c, b \to a.$