# XorSum

*-editorial-*

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*Task : Xor(1 <= i <= j <= n) (a[i] + a[j])*

***7 points :*** O(N^2) : Brute

***11 points :*** O(Vmax^2) : For every value x we count the number of its occurrences ( ap[x] ) in the given array. Now we take two values a <= b and we will have two cases :

- if a < b and every value occurs an odd number of times our answer will be updated with (a + b)
- if a = b the number of pairs (1 <= p <= q <= n and val[p] = val[q] = a) will be 1 + 2 + … + ap[a]. If this number is odd the answer will be updated with (a + b)

***32 points :*** O(Vmax * logVmax) : FFT

***27 points :*** O(N * logN * logVmax) : We will fix a bit , let call it B. We will get an array x i.e. x[i] = val[i] % $2^{B+1}$. We will sort this array x. If we get two values x[p] and x[q] , there will be 4 cases, more precisely, (x[p] + x[q]) will belong to an interval amongst the following ones : $I_1$ = [0… $2^B$ – 1] , $I_2$ = [$2^B$… 2 * $2^B$ - 1] , $I_3$ = [2 * $2^B$ … 3 * $2^B$ – 1] , $I_4$ = [3 * $2^B$ … 4 * $2^B$ – 1]. If the sum is in the second or in the fourth, the sum contains the bit B. For every x[pos] we will use binary search to find three indices p1 , p2 , p3, meaning that 1 <= i < p1 , x[i] + x[pos] ϵ $I_1$ ; p1 <= i < p2 x[i] + x[pos] ϵ $I_2$ ; p2 <= i < p3 x[i] + x[pos] ϵ $I_3$ and p3 <= i <= n x[i] + x[pos] ϵ $I_4$. If p2 – p1 + n + 1 – p3 is odd we will update the answer with $2^B$.

***23 points :*** O(N * logVmax) : It is the same idea like the previos one. The two essential observations are :

1. If we are at a bit B + 1, we can pass easily at bit B. This step involves that x[i] < $2^{B+2}$ We will split x in two parts. For every 1 <= i <= K , x[i] < $2^{B+1}$ and if K < i <= n , x[i] >= $2^{B+1}$. We will decrease every x[i] >= $2^{B+1}$ by $2^{B+1}$. These two parts are now sorted and we will merge them in O(n), resulting necessary x.
2. Now, for every position pos, we will update indices p1 , p2 , p3 advancing them from pos – 1 (two pointers trick).